

# Rapid Wireless Capacitor Charging Using a Multi-Tapped Inductively-Coupled Secondary Coil

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**Abstract**—This paper presents an inductive coupling system designed to wirelessly charge ultra-capacitors used as energy storage elements. Although ultra-capacitors offer the native ability to rapidly charge, it is shown that standard inductive coupling circuits only deliver maximal power for a specific load impedance which depends on coil geometries and separation distances. Since a charging ultra-capacitor can be modeled as an increasing instantaneous impedance, maximum power is thus delivered to the ultra-capacitor at only a single point in the charging interval, resulting in a longer than optimal charging time. Analysis of inductive coupling theory reveals that the optimal load impedance can be modified by adjusting the secondary coil inductance and resonant tuning capacitance. A three-tap secondary coil is proposed to dynamically modify the optimal load impedance throughout the capacitor charging interval. Measurement results show that the proposed architecture can expand its operational range by up to  $2.5\times$  and charge a 2.5 F ultra-capacitor to 5 V upwards of  $3.7\times$  faster than a conventional architecture.

**Index Terms**—Capacitor charging, inductive coupling, multi-tap inductor, ultra-capacitor, wireless power transfer.

## I. INTRODUCTION

NEAR-FIELD inductive wireless power transfer is an increasingly popular method to chronically deliver power to electronic devices that are otherwise difficult to interface with using direct wired connections. Applications are vast and include, among many others, implanted medical devices [1], [2], radio frequency identification (RFID) products [3], and non-contact semiconductor testers [4]. On the other hand, there are several application spaces where power is not delivered chronically, but is instead delivered over a finite amount of time, typically to periodically charge an energy storage element—for example wireless battery charging in consumer electronics, semi-active RFID, and electric vehicles [5]–[8].

Periodic charging of an energy storage element can in fact be a favorable alternative use-case for applications that tradi-

tionally employ chronic wireless power delivery. For example, biomedical implants could benefit greatly from both aesthetic and usability perspectives if the entire system can be fully implantable, eliminating the requirement of a semi-permanent external energy source. In this manner, a patient could retain the implant's functionality, even when performing tasks that are not conducive to wearing an external device (e.g., taking a shower or going swimming). However, the volume available to store energy within implants, portable consumer electronics, or electric vehicles is limited, and as a result, recharging must occur frequently. Unfortunately, batteries are slow to charge and can only do so a finite number of times, thereby requiring eventual, and potentially very expensive, replacement.

On the other hand, ultra-capacitors can support at least 1,000,000 re-charge cycles [9], potentially never requiring replacement over the useful lifetime of a device. The energy storage density of commercially available ultra-capacitors is, however, lower than batteries by approximately  $30\times$  [10], [11], although future research directions expect to decrease this gap significantly [12]. Additional power management is also required for devices utilizing ultra-capacitors as energy sources, in order to regulate the output voltage [13], [14]. The salient saving feature of ultra-capacitors as energy storage elements is their ability to rapidly charge. Unlike batteries, which can take hours to charge, ultra-capacitors can charge nearly instantly, limited only by power density or heat dissipation limits. From a user's perspective, convenient rapid charging can potentially offset the requirement of charging more frequently.

To this end, periodically charging an ultra-capacitor through an inductively coupled link presents a different problem formulation. Unlike chronic wireless power delivery systems which endeavor to maximize power transfer efficiency, the goal of this work is to instead *minimize charging time*. From a design perspective, this is equivalent to maximizing the amount of power delivered to the ultra-capacitor over transient conditions, given source and system constraints. Prior work in related areas focused either on resistive loads [15]–[17], utilized similar techniques to those used for resistive loads [18], or used alternative methods of ultra-capacitor charging that may not be suitable for all applications (e.g., solar [19]).

As will be shown in Section II, given a set of inductively coupled coil size and spacing geometries, a maximum power transfer condition occurs for a specific load impedance. This work presents an inductive coupling architecture that employs a multi-tapped secondary, as shown in Fig. 1, in order to dynamically modify the coil geometries, which in turn modify the optimal load impedance. This presents an opportunity to dynamically adjust the optimal load impedance to match the actual in-

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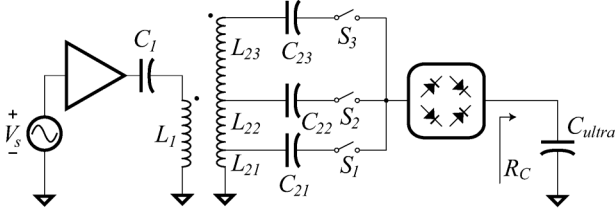


Fig. 1. Architecture of the multi-tapped secondary inductive coupling system.

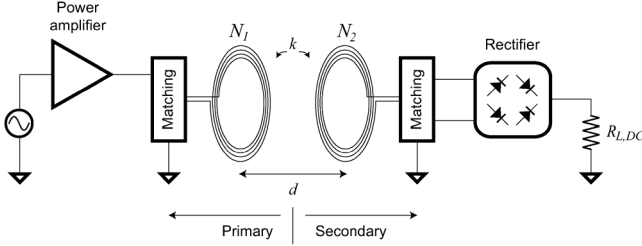


Fig. 2. Typical block diagram of an inductive coupling setup driving a resistive load.

stantaneous load impedance of the charging capacitor. Measurement results of a discrete prototype are shown in Section IV, and the paper concludes with a summary in Section V.

## II. INDUCTIVE COUPLING THEORY: MAXIMUM POWER TRANSFER

### A. Overview and Definitions

An illustration of a typical inductive coupling system is shown in Fig. 2. The circuit operates as follows: a power amplifier, with an RF voltage-source input, sends power through a primary-side coil with  $N_1$  turns. Matching is used to tune-out the inherent loop inductance in order to decrease loading effects on the power amplifier. So long as the operational wavelength is much less than the physical dimension of the coils and their separation, energy will be contained solely in near-field magnetics. Thus, the secondary-side coil, composed on  $N_2$  turns and spaced a distance  $d$  from the primary-side coil, receives energy from the transmitter's produced time-varying magnetic field. The RF output of the secondary-side matching network then passes through a rectifier, converting the AC energy into DC energy used to power the load, in this case modeled by resistor  $R_{L,DC}$ .

Since the coils are well-modeled by lumped inductors under near-field approximations, the matching networks typically consist of either series or parallel capacitors. Sizing these capacitors to resonate with their respective inductors at the frequency of power transfer has been shown to maximize efficiency and minimize loading of the power amplifier [15]. On the primary, series-tuning is typically employed for voltage-source drivers (i.e., most class-A through class-F power amplifiers). The choice of secondary-side tuning depends greatly on the application. It is well known that using a parallel secondary tuning capacitance induces a voltage multiplication factor, making rectifier diodes easier to turn-on. This makes a series-parallel link configuration ideal for low-power (i.e., high  $R_L$ ) applications, where it is

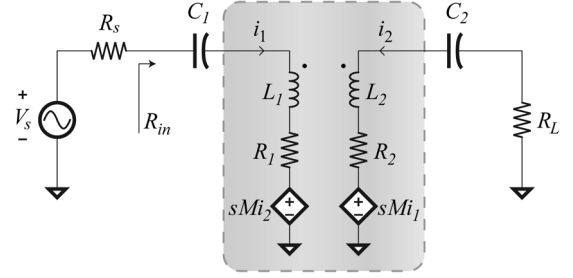


Fig. 3. Inductive coupling circuit model using series-series capacitive tuning.

difficult to generate sufficient voltage to activate non-linear rectifiers. At high powers (i.e., low  $R_L$ ), the voltage multiplication factor may present voltages that go far beyond CMOS or diode compatibility, limiting its utility. Instead, a series-series link is preferred for high power applications, where increased power is delivered through Q-multiplied current rather than voltage.

A more detailed circuit diagram of a typical inductive coupled system for high power applications is shown in Fig. 3. Inductors  $L_1$  and  $L_2$  model the primary and secondary loop reactances, respectively. The quality factor of each coil is given by  $Q_{1,2} = \omega L_{1,2}/R_{1,2}$ , where  $\omega$  is the operating frequency and  $R_{1,2}$  represents the parasitic series loss resistance. Voltage sources  $sMi_2$  and  $sMi_1$  model the mutual coupling effects between the coils, where  $M$  is the mutual coupling factor, while capacitors  $C_1$  and  $C_2$  provide resonant matching with the inductors. The coil coupling coefficient,  $k$ , is defined by the following equation:

$$k \equiv \frac{M}{\sqrt{L_1 L_2}}. \quad (1)$$

It can be shown that  $k$  is dependent only on coil separation distance  $d$  and individual coil geometries [15]. Importantly, it should be noted that  $k$  varies between 0 (no coupling) and 1 (perfect coupling), and decreases with  $d$ .

The primary side is driven by a voltage source,  $V_s$ , that has a series resistance,  $R_s$ . The secondary side is loaded by a resistor,  $R_L$ . Note that the actual load circuits are always found at the output of the rectifier. However, it can be easily shown that, with respect to the schematic in Fig. 2,  $R_L \approx R_{L,DC}/2$  [15]. Finally, an important definition that will be useful for analysis is the turns ratio of an ideal transformer, assuming perfect coupling:

$$n \equiv \sqrt{\frac{L_2}{L_1}}. \quad (2)$$

### B. Reflected Load Analysis

Inductive coupling theory has been well-studied in the past, and many excellent references describe analytical expressions that can accurately predict the maximum achievable efficiency of a given link [15]–[17]. However, the new problem formulation of maximizing power transfer requires new analysis to determine optimal design conditions. Building on existing work, this section derives new analytical results for predicting maximum power transfer conditions. Although a series-tuned secondary is considered here, the results can be easily extended to a parallel-tuned secondary by employing a parallel-to-series

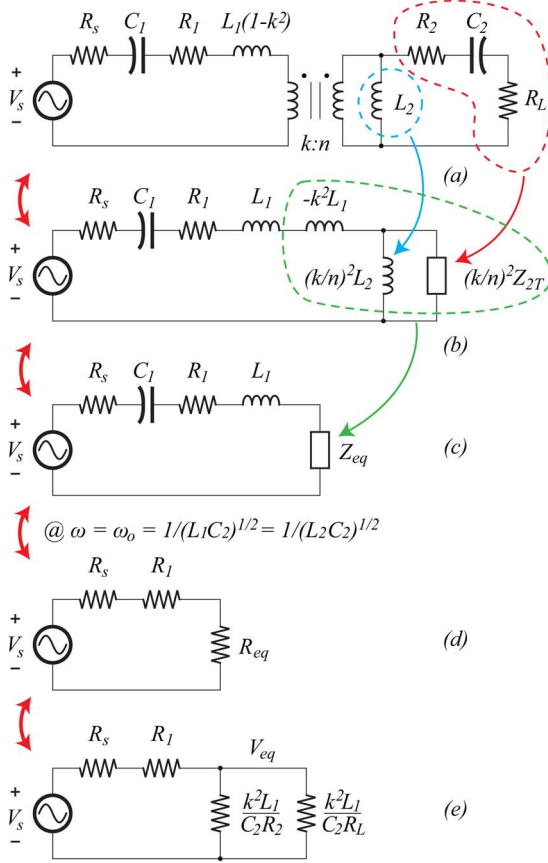


Fig. 4. Equivalent circuit models using reflected load analysis to determine analytical expressions for maximum power transfer.

impedance transformation [20]. It is important to note, however, that resonance can be achieved when converting a parallel load impedance,  $R_{L,p}$ , to a series equivalent,  $R_L$ , only if  $1/R_{L,p}^2 C_2^2 > 1/L_2 C_2$ . For low values of  $R_{L,p}$  (i.e., higher power applications), this will not always be the case; this is a further reason to use a series-tuned secondary. Otherwise, a primary circuit that senses this condition and shifts its driving frequency would be necessary to compensate for this effect [21].

In order to determine what conditions permit transfer of maximum power to the load, it is necessary to obtain workable expressions for simplified, though equivalent circuits. It is well known that the inductive coupling circuit in Fig. 3 can be modeled with an ideal transformer with turns ratio  $n/k$ , leakage inductance  $L_1(1 - k^2)$ , and magnetizing inductance  $L_2$ , as shown in Fig. 4(a). It is also well-known that circuit elements can be reflected across the terminals of an ideal transformer via a multiplicative factor of  $(k/n)^2$ . This is the basis for reflected load analysis. Similar analysis of the circuit in Fig. 2 can be performed using coupled-mode theory as generally preferred by physicists [22], though such analysis has been shown to be equivalent to reflected load analysis under near-field approximations with relatively low coupling coefficients and high quality factors [23]. Since reflected load analysis is valid over a larger range of assumptions, it is used as the main vehicle of analysis for this paper.

To gain design insight through simplified yet equivalent circuits, the total non-inductive secondary impedances in Fig. 4(a)

are grouped together such that they exist in parallel to  $L_2$  for reflection to the primary side. The total secondary impedance is given by  $Z_{2T} = R_2 + R_L + 1/j\omega C_2$  and is shown reflected to the primary side in Fig. 4(b).

Due to the parallel combination of an inductance and complex impedance, Fig. 4(b) is still too cumbersome for practical analysis. Mapping the values in Fig. 4(b) to Fig. 4(c) is the next step in enabling useful analysis. This can be done as follows:<sup>1</sup>

$$\begin{aligned} Z_{eq} &= -j\omega k^2 L_1 + \frac{1}{\frac{1}{j\omega(\frac{k}{n})^2 L_2} + \frac{1}{(\frac{k}{n})^2 Z_{2T}}} \\ &= -j\omega k^2 L_1 + \frac{1}{\frac{1}{j\omega k^2 L_1} + \frac{1}{(\frac{k}{n})^2 Z_{2T}}} \\ &= \frac{\omega^3 k^2 L_1 L_2 C_2 [\omega C_2 (R_L + R_2) + j(1 - \omega^2 L_2 C_2)]}{\omega^2 C_2^2 (R_L + R_2)^2 + (1 - \omega^2 L_2 C_2)^2}. \end{aligned} \quad (3)$$

However, Equation (3) is not very insightful for analysis. Fortunately, the expression simplifies considerably at resonance (i.e.,  $\omega = \omega_o = 1/\sqrt{L_2 C_2}$ )

$$Z_{eq}|_{\omega_o} = \frac{k^2 L_1}{C_2 (R_L + R_2)} = R_{eq}. \quad (4)$$

What is critical to note here is that this is a purely real impedance. The equivalent circuit under this resonant condition is shown in Fig. 4(d), assuming that  $L_1$  and  $C_1$  are also designed to resonate at the same frequency as the secondary (i.e.,  $\omega = \omega_o = 1/\sqrt{L_1 C_1} = 1/\sqrt{L_2 C_2}$ ), thereby shorting themselves out at the resonant frequency. The expression for  $R_{eq}$  can then be split into two different components as follows:

$$\begin{aligned} R_{eq} &= \frac{k^2 L_1}{C_2 (R_L + R_2)} \\ &= \frac{k^2 L_1}{C_2} \left( \frac{R_L R_2}{R_L + R_2} \right) \frac{1}{R_L R_2} \\ &= \left( \frac{k^2 L_1}{C_2 R_2} \right) // \left( \frac{k^2 L_1}{C_2 R_L} \right). \end{aligned} \quad (5)$$

A circuit with these two separated components is shown in Fig. 4(e). Given that there is an isolated component proportional to the load resistor, the power consumption seen at the load can be found using the following set of equations:

$$\begin{aligned} P_{out} &= V_{eq,rms}^2 \left( \frac{C_2 R_L}{k^2 L_1} \right) \\ V_{eq,rms} &= V_{s,rms} \left( \frac{R_{eq}}{R_s + R_1 + R_{eq}} \right) \\ &= V_{s,rms} \left( \frac{k^2 L_1}{C_2 (R_s + R_1) (R_L + R_2) + k^2 L_1} \right) \\ P_{out} &= V_{s,rms}^2 \left( \frac{k^2 L_1 C_2 R_L}{(k^2 L_1 + C_2 (R_s + R_1) (R_L + R_2))^2} \right). \end{aligned} \quad (6)$$

Solving the following equation can lead to the load impedance that maximizes power transfer, given all other inductive coupling conditions:

$$\frac{\partial P_{out}}{\partial R_L} = 0. \quad (7)$$

<sup>1</sup>This result was also shown in [17].

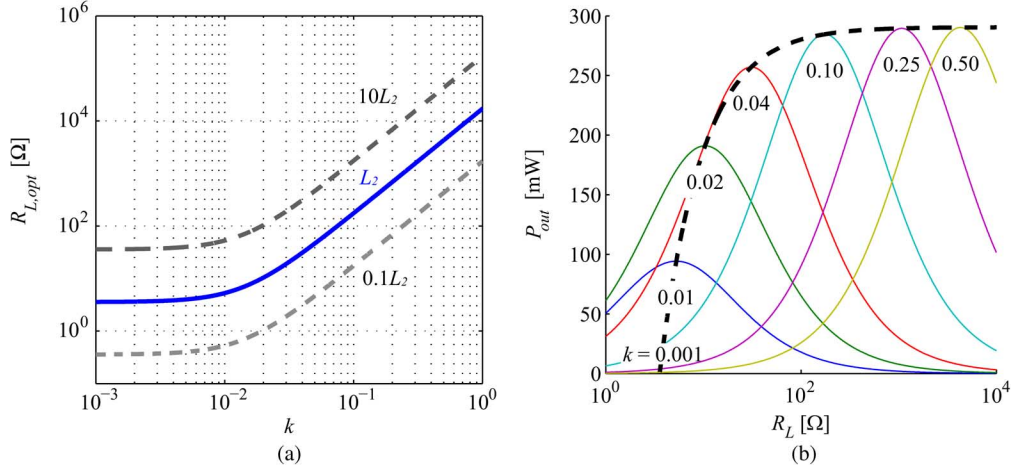


Fig. 5. Load impedances that maximize power transfer. In this example,  $V_s = 3.3 \text{ V}_{p-p}$ ,  $f = 6.78 \text{ MHz}$ ,  $L_1 = 3.3 \text{ } \mu\text{H}$ ,  $L_{2,norm} = 6.7 \text{ } \mu\text{H}$ ,  $Q_1 = 60$ , and  $Q_2 = 80$ . (a) Load impedances that maximize power transfer plotted across all practically useful coupling coefficients for various values of  $L_2$ . (b) Amount of extracted output power plotted across a range of load resistances for various values of  $k$  (whose values are annotated in plain text on the figure). The solid dashed line represents the maximum extractable power for a given  $k$ .

The solution to this equation is shown here

$$R_{L,opt} = R_2 + \frac{k^2 L_1}{C_2(R_s + R_1)}. \quad (8)$$

Lumping  $R_s$  together with  $R_1$  and substituting  $R$ 's for  $Q$ 's and  $L$ 's further simplifies the equation

$$\begin{aligned} R_{L,opt} &= \frac{\omega L_2}{Q_2} + \frac{k^2 Q_1}{\omega C_2} \\ &= \frac{1}{Q_2} \sqrt{\frac{L_2}{C_2}} + k^2 Q_1 \sqrt{\frac{L_2}{C_2}} \\ &= \sqrt{\frac{L_2}{C_2}} \left( \frac{1 + k^2 Q_1 Q_2}{Q_2} \right). \end{aligned} \quad (9)$$

Remarkably, this is a very similar to expression of load impedance that maximizes power transfer efficiency (see the Appendix for further details).

Equation (9) shows that the ideal load impedance (i.e., the value of  $R_L$  that maximizes power transfer to itself) changes with the coupling coefficient,  $k$ . It is also dependent on the ratio of secondary reactance values and the coil quality factors. Thus, the optimal impedance depends on the separation between coils, as well as their geometries. Fig. 5(a) plots the optimal load impedances across all practical values of  $k$  for the particular inductive parameters indicated in the figure caption. In this example,  $R_L$  varies from a minimum of  $3.6 \text{ } \Omega$  at  $k \approx 0$ , to a maximum of  $17 \text{ k}\Omega$  at  $k = 1$ . Interestingly, the optimal  $R_L$  at low coupling coefficients is exactly equal to the parasitic resistance of the secondary coil,  $R_2$ , as can be inferred by (8). This matches intuition well, since the state of the secondary circuit has almost no bearing on the primary circuit under low coupling conditions (i.e., its reflected impedance to the primary is negligible), and thus a maximum power matching circuit on the secondary is the configuration that extracts the most power. At higher levels of coupling, reflected impedances are no longer negligible. Thus, matching  $R_L$  to  $R_2$  would significantly load the primary circuit, thereby falling out of a maximum power transfer condition.

Fig. 5(b) shows the extractable output power from the same inductive coupling setup as described in Fig. 5(a), but this time plotted for varying output loads and coupling coefficients. Also shown is the maximum theoretically attainable output power and the associated load resistances for  $k$  varying from 0.001 to 0.5. At low coupling coefficients, the optimal  $R_L$  is small, as is the maximum extractable power. As  $k$  increases, the optimal  $R_L$  and associated extractable power increases. Although  $R_{L,opt}$  continues to increase with  $k$ ,  $P_{out,max}$  sees diminishing returns due to the fixed source impedance, eventually plateauing beyond coupling coefficients greater than 0.2. In other words, at high values of  $k$ , the reflection coefficient becomes large, requiring a large  $R_L$  (due to the inverse relationship) to create an impedance that matches  $R_1 + R_s$ . At this point, the reflected  $k^2 L_1 / (C_2 R_2)$  becomes very large relative to  $k^2 L_1 / (C_2 R_L)$ , making its effects negligible. Thus, the maximum power transfer theory can be simply applied here:  $k^2 L_1 / (C_2 R_L) = R_1 + R_s$ , resulting in a maximum of 50% power transfer efficiency.

### C. Charging Capacitor Model

AC to DC converters employ the use of non-linear elements such as diodes to perform rectification. This makes rectifier modeling challenging, as large-signal analysis is typically required. Although charging a capacitor is by definition a transient event, it is possible to generate a model using a quasi-steady-state approximation. Specifically, a charging capacitor can be modeled over a sufficiently short period of time (given the size of the capacitor and input current) as a DC voltage source. At steady-state, a rectifier feeding a DC voltage source can be modeled as a resistor whose resistance depends on the input voltage, output voltage, and diode characteristics [24]. Thus, a charging capacitor can be modeled as a resistor,  $R_C$ , whose resistance instantaneously changes as the voltage on the capacitor increases.

### III. MULTI-TAPPED SECONDARY DESIGN

#### A. Theory

A capacitor whose instantaneous resistance,  $R_C$ , changes with its increasing output voltage is an issue when using an inductively-coupled charging system: since there is an optimum impedance that maximizes power transfer to the load, there is at most only a single point in time during the capacitor charging interval at which it is charging at the maximum possible rate (i.e., when  $R_C = R_{L,opt}$ ). To overcome this issue and provide maximum power for larger portions of the charging time, a proposed solution could alter the effective load impedance using an additional matching network (e.g., [25]) or an output DC/DC converter. However, given finite passive quality factors, it is difficult to achieve large impedance transformation ratios at high efficiencies.

Instead, (9) shows that the optimal impedance depends on the ratio of secondary reactances:  $\sqrt{L_2/C_2}$ . Under resonant conditions,  $L_2$  and  $C_2$  are naturally related by the following relation:  $\omega_o = 1/\sqrt{L_2 C_2}$ . Thus, the optimal impedance for maximum power transfer shown in (9) can be re-written in terms of either  $C_2$  or  $L_2$ , the latter of which is shown here

$$R_{L,opt} = \omega_o L_2 \left( \frac{1 + k^2 Q_1 Q_2}{Q_2} \right). \quad (10)$$

Changing the value of  $L_2$  (and, correspondingly,  $C_2$ ) can therefore change  $R_{L,opt}$  for a given  $k$ , as shown in Fig. 5(a). Such a circuit can, at any point in time and for any  $k$ , change  $R_{L,opt}$  to equal the charging capacitor's instantaneous resistance ( $R_C$ ) at that precise instance in time. This secondary reactance modulation technique can be performed dynamically to provide robustness to instantaneously changing load impedances, or it can be performed semi-statically to provide robustness against variations in coil separation distances, all while decreasing charging time. Although tunable inductors are often difficult to implement, others have described and validated such circuits for RFIC applications using interesting bridge topologies [26] or multi-tap structures [27].

#### B. Implementation

1) *Architecture*: The architecture of the proposed multi-tap-secondary inductive coupling system for rapid wireless capacitor charging is shown in Fig. 1, and a photograph and schematic of the testing setup is shown in Fig. 6. The secondary coil is designed as a single large coil with inductance  $L_{23}$ . Two smaller inductances,  $L_{22}$  and  $L_{21}$ , are created by tapping into fewer turns of the coil. Each tap is allocated a single series capacitor used to resonate with the effective inductance seen at the output of the tap. The output of the series capacitors are connected to series switches, used to select a single tap configuration at a time.

The number of taps and turns selected for each additional tap can be selected at design time by considering the range of expected values of  $R_L$  and  $k$ , and ensuring that adequate power transfer is achieved over this range. For example, the design recipe described in [17], together with the equations presented in this paper for maximum power transfer (rather than efficiency), can be used as a starting point to analytically determine the

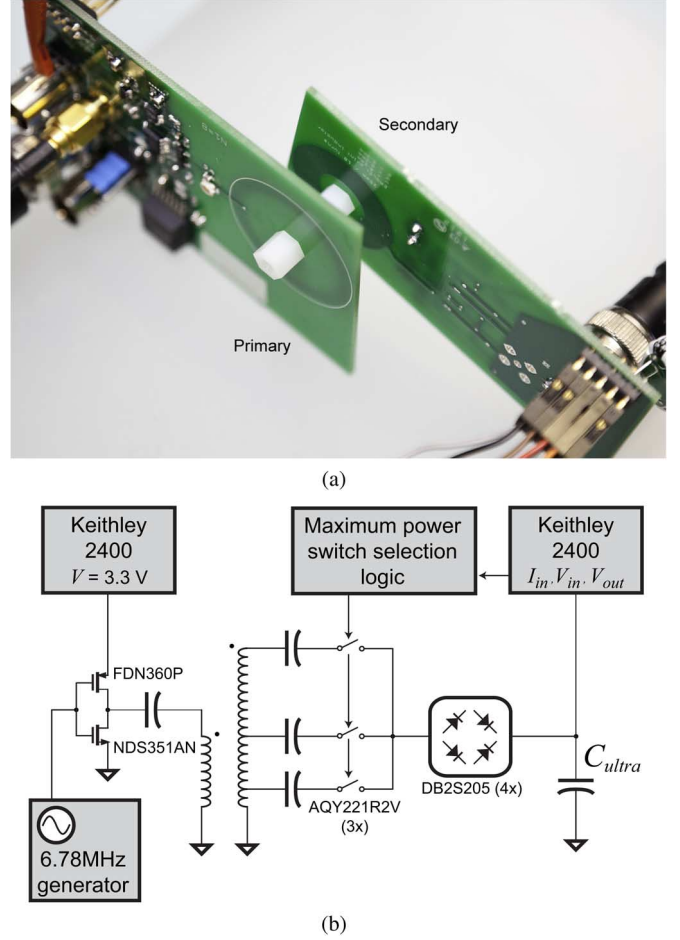


Fig. 6. Photograph (a) and schematic (b) of the testing setup for the rapid wireless capacitor charger.

desired inductances, given  $R_L$  and  $k$  bounds. Electromagnetic simulation or experimentation can then be used to refine tap selections.

To have minimal effect on the inductor quality factors, the series tap-selection switches should be designed with as low on-resistance as possible. However, low-impedance switches typically have large associated parasitic capacitances. Additionally complicating the matter is the need for high voltage-blocking capabilities due to the (loaded) Q voltage multiplication on unused taps. The best trade-off here is to select a switch whose impedance is minimal, while offering sufficient voltage blocking capability and parasitic capacitance that does not overwhelm or appreciable change the resonant frequency set by the series capacitors.

As discussed in more detail in the following subsection, the discrete prototype design employs the use of a secondary inductor that is 25 mm in diameter. At this size, a total of 20 turns achieves an approximate inductance of less than 10  $\mu\text{H}$ . At the operational frequency of 6.78 MHz used throughout this paper, such an inductor size results in a minimum series tuning capacitance of approximately 50 pF. For a proof-of-concept prototype, a Panasonic AQY221R2V solid-state optoelectronic relay was chosen as the switching element, as its on-resistance is 0.75  $\Omega$ , its parasitic capacitance is 12.5 pF, and it can safely block 40



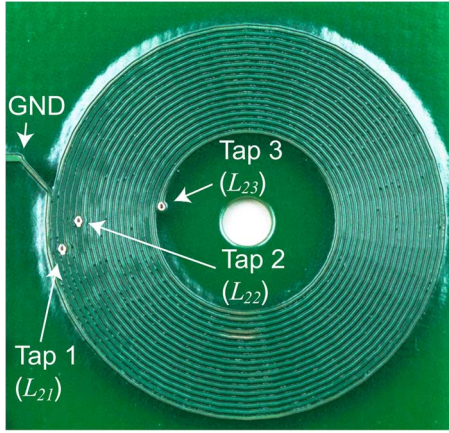


Fig. 7. Photograph of the three-tap secondary coil.

V. When turned on, the relay requires approximately 5 mW to operate.<sup>2</sup>

Many inductively-coupled applications employ the use of class-E power amplifiers on the primary circuit for high efficiency [28]. However, a class-E amplifier requires very precise knowledge of the load impedance in order to operate properly (and therefore at high efficiency) [20]. As previously discussed, inductive coupling systems operating with varying  $k$  and  $R_C$  conditions can present wildly varying impedances as seen by the primary, making the design of uncompensated class-E amplifiers impractical for these cases. To combat this issue, others have described interesting control loops to provide robustness against load variations that offer promising potential [15]. On the other hand, class-D amplifiers can operate reasonably efficiently without significant regard for load impedances. Consequently, such an amplifier was chosen for this initial prototype. Specifically, the amplifier is implemented as an inverter structure with NDS351AN and FDN360P nMOS and pMOS transistors, respectively.

The rectifier is composed of four Panasonic DB2S205 Schottky diodes in a bridge configuration.

2) *Coil Design:* The primary and secondary coils in the discrete prototype are both designed as printed inductors on an FR-4 substrate circuit board. The primary coil is an  $N = 8$  turn design, while the secondary coil is an  $N = 18$  turn design. In this particular implementation, it was found that two additional taps (at turns 2 and 5) offered sufficient power transfer across the range of expected  $k$  (0.05–0.3) and  $R_L$  (10–1000  $\Omega$ ). A photograph of the implemented secondary coil is shown in Fig. 7. Both the primary and secondary coils are printed using 2-oz copper with a trace width/spacing of 0.2 mm. The primary coil is designed with a diameter of 30 mm, while the secondary coil is designed to be slightly smaller at 25 mm for robustness to mis-alignment.

Electromagnetic simulations were performed using Mentor Graphics IE3D to extract inductance and mutual coupling infor-

TABLE I  
SIMULATED COIL PARAMETERS

	$L_1$	$L_{21}$	$L_{22}$	$L_{23}$
$N$	8	2	5	18
$L$ [ $\mu$ H]	3.31	0.28	1.25	6.73
$R_{series}$ [ $\Omega$ ]	2.02	0.47	1.10	3.07
$Q$	70	26	48	93
$C_{tune}$ [pF]	167	1954	443	82

mation. A summary of the simulated coil parameters are shown in Table I.

#### IV. MEASUREMENT RESULTS

The inductive coupling system was tested in a typical electronics lab environment using nylon board-spacers of various lengths to separate the primary and secondary coils. Voltage and power monitoring were simultaneously performed using a Keithley 2400 sourcemeter at the output of the rectifier.

Fig. 8 demonstrates the benefits of using a multi-tapped secondary coil by measuring received output power: not a single tap configuration offers the best performance across various coil separation distances and output capacitor voltages. At close coil separation distances (i.e., high- $k$ ), inductor  $L_{21}$  delivers the most power to the load, while inductor  $L_{23}$  delivers the most power to the load at long distances (low- $k$ ). At intermediate distances, dynamically switching between secondary coil configurations as the output capacitor voltage increases can achieve superior power delivery results. The thin lines in Fig. 8 are measurement results without using the series switches (i.e., a direct connection is instead used). At worse, the series switches add a 9.7% power loss at maximum power transfer (this includes the 5 mW switch operation power).

The  $x$  axis shown in Fig. 8 is converted to resistance values by dividing the output capacitor voltage by its incoming current; these results are then compared in Fig. 9 to analytical predictions. Measured results match the theoretically predicted behavior very closely for all tap configurations, with the exception of the third tap at long coupling distances, likely due to a coupling coefficient estimation mismatch. Note that the measured output resistance is limited under certain tap and distance configurations due to output capacitor voltage limits.

The implemented inductive coupling circuit was used to demonstrate improved capacitor charging time and range. Fig. 10 shows measured transient results of charging a 2.5 F ultra-capacitor to 5 V for coil separation distances of 9.5 mm and 19.2 mm. The thin lines correspond to static individual tap configurations, while the thick lines correspond to dynamic switching between taps. Measurements of static individual tap configurations were obtained by removing the series switches (i.e., shorting out a direct connection) to better compare the proposed architecture to conventional approaches.<sup>3</sup>

In these experiments, a Keithley 2400 sourcemeter was employed as a periodic power meter to determine the optimal inductor tap selection, as shown in Fig. 6(b). Specifically, the sourcemeter was connected directly across the ultra-capacitor,

<sup>2</sup>Future work involving switch integration and/or miniaturization may require the use of a process compatible with high voltages (for example thick-oxide devices in CMOS, or GaN or SiC processes) for voltage blocking capabilities. Alternatively, it may be possible to integrate the relay bare die into a system-in-package, or employ the use of MEMS-based relays instead.

<sup>3</sup>Due to a measurement setup limitation, the 5 mW quiescent switch power was not taken into account for charging-time measurements; the lost power consumption was indeed included for power transfer measurements, however.

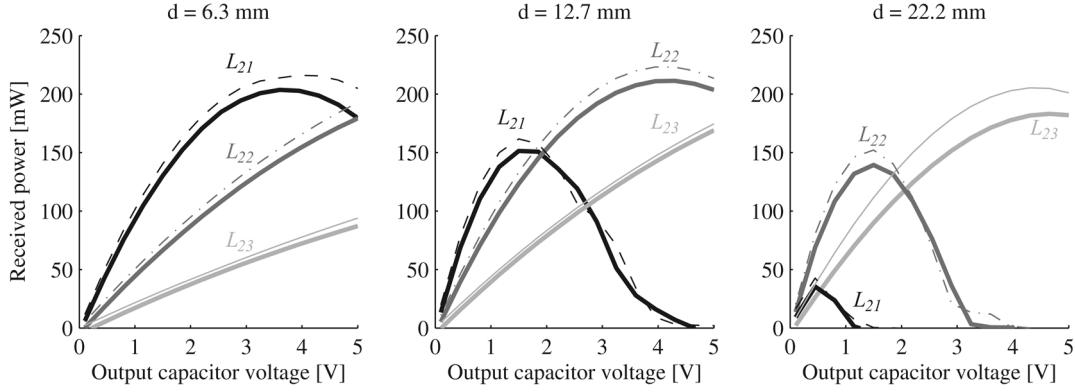


Fig. 8. Power delivered to a Keithley 2400 sourcemeter set at fixed voltage values for a few select distances. The thin curves correspond to a fixed secondary tap configuration (i.e., without any series switches). Here,  $V_s = 3.3 V_{p-p}$  and  $f = 6.78$  MHz.

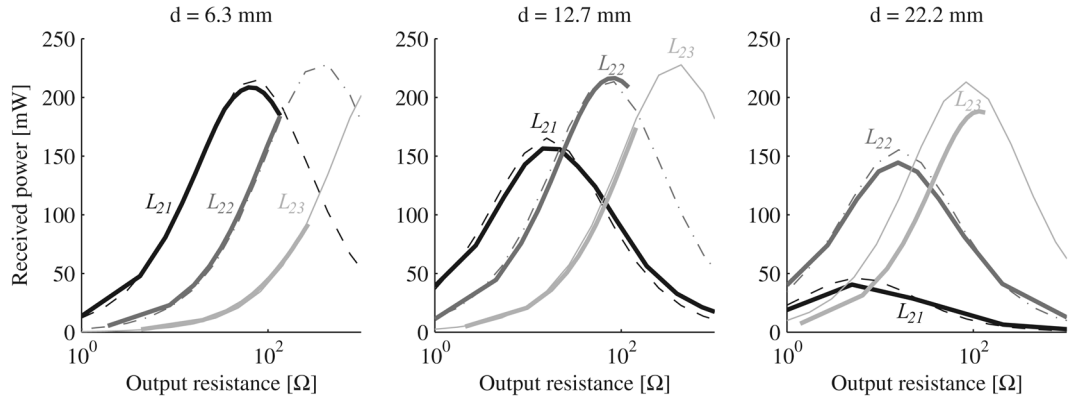


Fig. 9. Measured power delivery compared to analytically predicted power delivery in thick and thin curves, respectively. The abrupt ending of measured curves occur when the capacitor reaches an output voltage of 5 V. Here,  $V_s = 3.3 V_{p-p}$  and  $f = 6.78$  MHz under measurement conditions, and  $L_1 = 3.3 \mu\text{H}$ ,  $L_{21} = 0.28 \mu\text{H}$ ,  $L_{22} = 1.25 \mu\text{H}$ ,  $L_{23} = 6.73 \mu\text{H}$ ,  $Q_{1,eff} = 48$ ,  $Q_{21,eff} = 2.5$ ,  $Q_{22,eff} = 10$ , and  $Q_{23,eff} = 40$  for analytical calculations (effective  $Q$ s lump in  $R_1$ , series switch resistance, and rectifier losses).

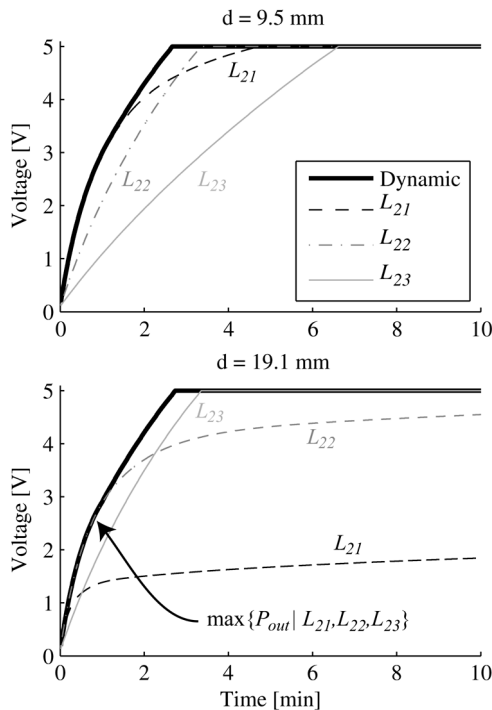


Fig. 10. Transient measurement of charging a 2.5 F ultra-capacitor to 5 V. The thin lines corresponds to exclusive use of inductors  $L_{21}$ ,  $L_{22}$ , and  $L_{23}$ , while the thick black line corresponds to dynamic switching between them.

while most often operating in a high-impedance voltage-monitoring mode. Periodically, when the output voltage crossed 0.5 V charging thresholds, the sourcemeter would temporarily enter a low-impedance voltage-sourcing, current-sensing mode, while the tap configurations were iterated through (the voltage was set to exactly the current ultra-capacitor state). The tap corresponding to the highest level of received power was selected for operation until the next 0.5 V threshold was reached. In this manner, the circuit was always configured to receive maximum power, so as to charge the load capacitor as rapidly as possible, regardless of any losses in up-stream components. Alternatively, it is instead possible to directly measure the instantaneous load impedance in order to match it to the theoretical optimum. However, such an approach would likely require a series sensor, which would not take into account the impedance of up-stream components (the rectifier, for example), while also introducing additional losses. Although the proposed mode of control employing an external sourcemeter is feasible for discrete prototyping, it is not suitable for an IC implementation. Rather than implementing a full power meter, future work on an IC design could instead monitor how rapidly the ultra-capacitor (or a smaller test capacitor) voltage rises for the various tap configurations, thereby directly indicating which tap is optimal for rapid charging.

The charging time and power transfer results for various coil separation distances are summarized in Fig. 11 and Table II. In general, inductor  $L_{21}$  provides the best performance amongst

TABLE II  
SUMMARY OF CHARGING TIME AND RESISTIVE SWITCH LOSSES FOR THE DISCRETE PROTOTYPE SYSTEM

Coil separation distance [mm]	Charging time [minutes]				Maximum charging time improvement <sup>†</sup>	Peak power delivery <sup>‡</sup> [mW]	Maximum series switch resistive loss + quiescent loss at peak power delivery
	$L_{21}$	$L_{22}$	$L_{23}$	Dynamic <sup>†</sup>			
4.4	3.3	6.0	12.4	3.4	3.7X	198 (tap 1)	14.3 + 5.0 = 19.3 mW (9.7%)
6.3	2.9	4.5	9.7	2.8	3.5X	203 (tap 1)	5.3 + 5.0 = 10.3 mW (5.1%)
9.5	4.7	3.3	6.6	2.7	2.4X	216 (tap 2)	6.4 + 5.0 = 11.4 mW (5.3%)
12.7	26.6	2.8	4.8	2.6	10.2X	211 (tap 2)	6.7 + 5.0 = 11.7 mW (5.5%)
19.1	$\infty^*$	22.6	3.3	2.7	$\infty^* / 8.3X$	212 (tap 3)	4.1 + 5.0 = 9.1 mW (4.3%)
22.2	$\infty^*$	$\infty^*$	3.6	3.1	$\infty^* / 1.2X$	183 (tap 3)	16.6 + 5.0 = 21.6 mW (9.1%)

<sup>†</sup> Includes resistive switch losses, but not quiescent switch power consumption losses.

<sup>‡</sup> Includes all switch losses.

\* At such coil separations, it is, for all practical purposes, not possible to charge to 5 V. Charging time-improvements in these cases are for the next-best tap configuration.

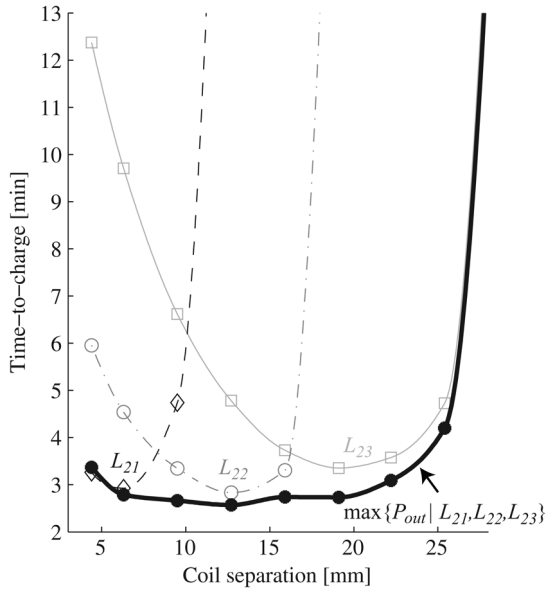


Fig. 11. Charging time for a 2.5 F, 5 V ultra-capacitor. The thin lines corresponds to exclusive use of inductors  $L_{21}$ ,  $L_{22}$ , and  $L_{23}$ , while the thick black line corresponds to dynamic switching between them.

static configurations at small coil separation distances, inductor  $L_{22}$  at medium distances, and inductor  $L_{23}$  at long distances. Meanwhile, dynamically switching between all three taps provides superior charging times at nearly all coil separation distances. For example, at a distance of 4.4 mm, dynamic switching achieves a charging time that is  $3.7\times$  faster than using inductor  $L_{23}$  alone.<sup>4</sup> At a distance of 12.7 mm, however, inductor  $L_{21}$  requires 26 minutes to reach 5 V, which is  $10.2\times$  slower than the dynamic configuration. Similarly, at a distance of 19.1 mm, inductor  $L_{21}$  can no longer charge the load to 5 V, while the dynamic system requires 2.7 minutes—which is  $8.3\times$  faster than using inductor  $L_{22}$  in a static configuration. As a result of the dynamic inductor reconfigurability, the operational range of the system whereby the ultra-capacitor is charged to 5 V in a reasonable amount of time is expanded by upwards of  $2.5\times$ .

## V. CONCLUSIONS

The goal of this work was to provide, through an inductively coupled link, maximum power to a load capacitor in order to charge it as rapidly as possible. Although analysis

<sup>4</sup>Interestingly, operating inductor  $L_{21}$  alone at 4.4 mm offers fractionally faster charging time than the dynamic configuration, due to reduced resistive losses (owing to the removed series switch for static tap testing).

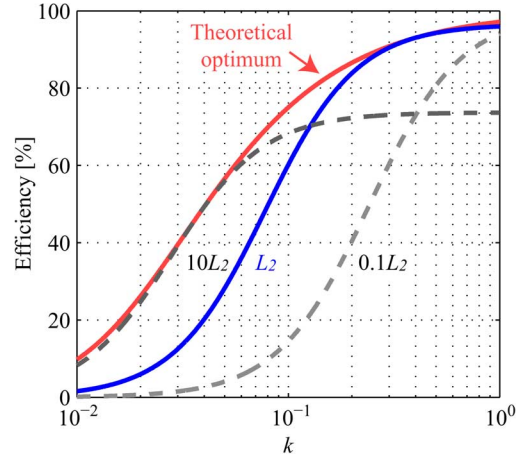


Fig. 12. Wireless power transfer efficiency under the same conditions as shown in Fig. 5, for a load impedance  $R_L = 100 \Omega$  and various values of  $L_2$ . The solid red curve is the theoretically maximal efficiency under optimal conditions for each value of  $k$ .

of inductive coupling theory is conventionally performed to optimize efficiency or link gain, this paper instead presented analysis used to maximize power transfer to the load. It was found that given coil geometries and separation distances, there exists a load resistance that maximizes power transfer. Since a charging capacitor can be modeled as an instantaneously increasing impedance, there exists at most one point in time during the capacitor charging interval where maximum power is being delivered. However, it was also found that the optimal load impedance can be modified by adjusting the ratio of secondary reactances. As a result, this paper presented a multi-tapped secondary coil whose inductance and resonant tuning capacitance could be dynamically adjusted in order to deliver maximum power to the load over a wider range of output impedances and coil separation distances. Measurement results verified the feasibility of this concept, demonstrating an inductive link that operated over a  $2.5\times$  longer range while charging an ultra-capacitor up to several times faster than conventional approaches.

Although the analysis and experimental results were optimized for maximizing power transfer, the efficiency of an inductive link also has a maximum at a single optimal load impedance that depends on the ratio of secondary reactances (as shown in the Appendix). It could thus be possible for future work to extend the multi-tapped architecture to systems that require high efficiency over variable load or coil separation conditions.



# APPENDIX EFFICIENCY ANALYSIS

The maximum power transfer analysis developed in Section II-B can be leveraged to also derive optimal efficiency criteria. Total link efficiency can be broken out into two separate components: primary and secondary efficiencies. Specifically, the circuits in Fig. 4(d) and 4(a) can be used to calculate the primary and secondary efficiencies at resonance, respectively

$$\begin{aligned}\eta_{tot} &= (\eta_1) \times (\eta_2) \\ &= \left( \frac{R_{eq}}{R_{eq} + R_s + R_1} \right) \times \left( \frac{R_L}{R_L + R_2} \right) \\ &= \dots \left( \text{lumping } R_s \text{ together with } Q_1, \rightarrow Q_1 = \frac{\omega L_1}{R_1 + R_s} \right) \\ &= \frac{k^2 Q_1 \omega C_2 R_L}{\left( \omega C_2 R_L + k^2 Q_1 + \frac{1}{Q_2} \right) \left( \omega C_2 R_L + \frac{1}{Q_2} \right)}.\end{aligned}\quad (11)$$

Taking the derivative of efficiency with respect to  $R_L$  and equating to zero will yield the load impedance that maximizes power transfer efficiency

$$R_{L,\eta_{opt}} = \sqrt{\frac{L_2}{C_2}} \left( \frac{\sqrt{1 + k^2 Q_1 Q_2}}{Q_2} \right).\quad (12)$$

Similar results were also derived in [15], [17]. For completeness, the optimal efficiency given this result is shown here

$$\eta_{opt} = \frac{k^2 Q_1 Q_2}{(1 + \sqrt{1 + k^2 Q_1 Q_2})^2}.\quad (13)$$

What is very interesting to note here is that the  $R_{L,\eta_{opt}}$  expression is identical to the  $R_{L,opt}$  expression for maximum power transfer, with the only difference being the square root of  $(1 + k^2 Q_1 Q_2)$  in the numerator.

Since the optimal load impedance for efficiency is also dependent on the ratio of secondary reactances, the multi-tap architecture presented in this paper could also be used to improve efficiency over a wider range of output impedances or coupling coefficients. For example, Fig. 12 shows the efficiency of wireless power transfer for various  $k$  and  $L_2$ , given a fixed load impedance. It can be seen that at a given coupling coefficient, one of the three taps will offer superior power transfer efficiency. Naturally, such a technique could be combined with a primary-side supply modulation technique to deliver the appropriate amount of power at the highest possible efficiency [29].

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